

# A Statistical Model for the Joint Inference of Vertical Stability and Horizontal Diffusibility of Typological Features: Supplementary Material

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## S.1 Derivation of Eq. (2)

First, we expand the conditional probability distribution:

$$P(x_{l,i} = k \mid \mathbf{x}_{-l,i}, v_i, h_i, \mathbf{u}_i) = \frac{P(\{x_{l,i} = k\} \cup \mathbf{x}_{-l,i} \mid v_i, h_i, \mathbf{u}_i)}{\sum_{k'} P(\{x_{l,i} = k'\} \cup \mathbf{x}_{-l,i} \mid v_i, h_i, \mathbf{u}_i)}.$$

Applying Eq. (1) to both the numerator and the denominator and canceling out the common denominator, we have

$$\begin{aligned} P(x_{l,i} = k \mid \mathbf{x}_{-l,i}, v_i, h_i, \mathbf{u}_i) \\ = \frac{\exp\left(v_i V(\{x_{l,i} = k\} \cup \mathbf{x}_{-l,i}) + h_i H(\{x_{l,i} = k\} \cup \mathbf{x}_{-l,i}) + \sum_j u_{i,j} U_j(\{x_{l,i} = k\} \cup \mathbf{x}_{-l,i})\right)}{\sum_{k'} \exp\left(v_i V(\{x_{l,i} = k'\} \cup \mathbf{x}_{-l,i}) + h_i H(\{x_{l,i} = k'\} \cup \mathbf{x}_{-l,i}) + \sum_j u_{i,j} U_j(\{x_{l,i} = k'\} \cup \mathbf{x}_{-l,i})\right)}. \end{aligned}$$

Recall that  $\exp(a + b) = \exp(a)\exp(b)$  and that  $x_{l,i}$  only affects the counts concerning  $x_{l,i}$  and its neighbors. For example,  $V(\{x_{l,i} = k\} \cup \mathbf{x}_{-l,i})$  can be decomposed into  $V_{l,i,k}$  and a common factor. Canceling out the common factors, we obtain

$$\begin{aligned} P(x_{l,i} = k \mid \mathbf{x}_{-l,i}, v_i, h_i, \mathbf{u}_i) &= \frac{\exp(v_i V_{l,i,k} + h_i H_{l,i,k} + u_{i,k})}{\sum_{k'} \exp(v_i V_{l,i,k'} + h_i H_{l,i,k'} + u_{i,k'})} \\ &\propto \exp(v_i V_{l,i,k} + h_i H_{l,i,k} + u_{i,k}). \end{aligned}$$

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<sup>\*</sup>The work of the second author was done when he was a student in the Graduate School of Informatics, Kyoto University.

## S.2 Gibbs Sampling

Recall that we want to sample  $v_i$ ,  $h_i$ ,  $u_i$  and  $x_i^{\text{mis}}$  from  $P(v_i, h_i, u_i, x_i^{\text{mis}} \mid x_i^{\text{obs}}) \propto P(v_i)P(h_i)P(u_i)P(x_i \mid v_i, h_i, u_i)$ . Gibbs sampling (Bishop, 2006, Chapter 11) repeatedly updates each of  $v_i$ ,  $h_i$ ,  $u_{i,j}$  and  $x_{l,i} \in x_i^{\text{mis}}$  while the rest is fixed. Starting from some initial state, the updates are performed in random order. One sweep of the variables is counted as an iteration.

The pseudo-code is given in Algorithm 1. We have already derived the conditional probability of  $x_{l,i}$  in Section S.1. The updates of  $v_i$ ,  $h_i$  and  $u_{i,j}$  are more complicated, as we will see in the next section.

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**Algorithm 1** One iteration of Gibbs sampling

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- 1: **for all**  $v \in \{v_i, h_i, u_{i,1}, \dots, u_{i,F_i}\} \cup x_i^{\text{mis}}$  in random order **do**
  - 2:     **if**  $v \in x_i^{\text{mis}}$  **then**
  - 3:         Update  $v$  according to Eq. (2)
  - 4:     **else**
  - 5:         Update  $v$  as described in Section S.3
  - 6:     **end if**
  - 7: **end for**
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## S.3 Approximate Sampling from Doubly-intractable Distributions

During inference, we want to sample  $v_i$  (and  $h_i$  and  $u_{i,j}$ ) from its posterior distribution,  $P(v_i \mid h_i, u_i, x_i) \propto P(v_i)P(x_i \mid v_i, h_i, u_i)$ . Unfortunately, we cannot apply the standard Metropolis-Hastings (MH) sampler to this problem because  $P(x_i \mid v_i, h_i, u_i)$  contains an intractable normalization term. Such a distribution is called a *doubly-intractable* distribution because MCMC itself approximates the intractable distribution (Møller et al., 2006; Murray et al., 2006). This problem remains an active topic in the statistics literature to date. However, if we give up theoretical rigorousness, it is not difficult to draw samples from the posterior, which are only approximately correct but work well in practice.

Specifically, we use the double MH sampler (Liang, 2010). The key idea is to use an auxiliary variable to cancel out the normalization term. This sampler is based on the exchange algorithm of Murray et al. (2006), which samples  $v_i$  in the following steps.

1. Propose  $v'_i \sim q(v'_i \mid v_i, x_i)$ .
2. Generate an auxiliary variable  $y_i \sim P(y_i \mid v'_i, h_i, u_i)$  using an exact sampler.
3. Accept  $v'_i$  with probability  $\min\{1, r(v_i, v'_i, y_i \mid x_i)\}$ , where

$$r(v_i, v'_i, y_i \mid x_i) = \frac{P(v'_i)q(v_i \mid v'_i, x_i)}{P(v_i)q(v'_i \mid v_i, x_i)} \times \frac{P(x_i \mid v'_i, h_i, u_i)P(x_i \mid v_i, h_i, u_i)}{P(x_i \mid v_i, h_i, u_i)P(y_i \mid v'_i, h_i, u_i)}.$$

A problem lies in the second step. The exact sampling of  $y_i$  is as difficult as the original problem. Thus the double MH sampler approximates it with a Gibbs sampling scan of  $x_{l,i}$ 's starting from the current  $x_i$ . At each step of the Gibbs sampling scan,  $x_{l,i}$  is updated according to  $P(x_{l,i} \mid x_{-l,i}, v'_i, h_i, u_i)$ .

We construct the proposal distributions  $q(v'_i | v_i, \mathbf{x}_i)$  and  $q(h'_i | h_i, \mathbf{x}_i)$  using a log-normal distribution, and  $q(u'_{i,j} | u_{i,j}, \mathbf{x}_i)$  using a Gaussian distribution.

## References

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Table S.1: Features ranked by accuracy of missing value imputation (Autologistic-Basic).

	Feature	Accuracy (%)
143G	Minor morphological means of signaling negation	99.24
11A	Front Rounded Vowels	93.40
90C	Postnominal relative clauses	93.38
18A	Absence of Common Consonants	88.69
130A	Finger and Hand	87.82
73A	The Optative	84.91
6A	Uvular Consonants	82.86
111A	Nonperiphrastic Causative Constructions	82.20
93A	Position of Interrogative Phrases in Content Questions	80.47
82A	Order of Subject and Verb	79.68
19A	Presence of Uncommon Consonants	79.15
89A	Order of Numeral and Noun	78.99
86A	Order of Genitive and Noun	77.08
85A	Order of Adposition and Noun Phrase	76.90
83A	Order of Object and Verb	75.96
27A	Reduplication	75.62
87A	Order of Adjective and Noun	74.58
7A	Glottalized Consonants	72.08
88A	Order of Demonstrative and Noun	71.22
90A	Order of Relative Clause and Noun	70.23
119A	Nominal and Locational Predication	69.29
95A	Relationship between the Order of Object and Verb and the Order of Adposition and Noun Phrase	68.62
8A	Lateral Consonants	68.37
120A	Zero Copula for Predicate Nominals	67.72
44A	Gender Distinctions in Independent Personal Pronouns	66.84
52A	Comitatives and Instrumentals	66.36
55A	Numeral Classifiers	64.82
107A	Passive Constructions	64.50
129A	Hand and Arm	64.23
81A	Order of Subject, Object and Verb	62.94
101A	Expression of Pronominal Subjects	61.55

Table S.1: (continued)

	Feature	Accuracy (%)
116A	Polar Questions	61.32
51A	Position of Case Affixes	60.78
94A	Order of Adverbial Subordinator and Clause	60.33
91A	Order of Degree Word and Adjective	59.58
46A	Indefinite Pronouns	59.51
5A	Voicing and Gaps in Plosive Systems	59.19
69A	Position of Tense-Aspect Affixes	59.03
13A	Tone	58.37
96A	Relationship between the Order of Object and Verb and the Order of Relative Clause and Noun	58.31
143E	Preverbal Negative Morphemes	58.13
64A	Nominal and Verbal Conjunction	57.81
57A	Position of Pronominal Possessive Affixes	57.38
15A	Weight-Sensitive Stress	56.54
100A	Alignment of Verbal Person Marking	56.38
12A	Syllable Structure	56.29
33A	Coding of Nominal Plurality	55.68
144B	Position of negative words relative to beginning and end of clause and with respect to adjacency to verb	55.59
97A	Relationship between the Order of Object and Verb and the Order of Adjective and Noun	55.13
48A	Person Marking on Adpositions	54.81
77A	Semantic Distinctions of Evidentiality	54.68
143F	Postverbal Negative Morphemes	54.50
84A	Order of Object, Oblique, and Verb	54.31
9A	The Velar Nasal	54.27
112A	Negative Morphemes	54.07
72A	Imperative-Hortative Systems	53.74
70A	The Morphological Imperative	53.59
16A	Weight Factors in Weight-Sensitive Stress Systems	52.52
2A	Vowel Quality Inventories	52.04
102A	Verbal Person Marking	51.60
124A	'Want' Complement Subjects	50.88

Table S.1: (continued)

	Feature	Accuracy (%)
92A	Position of Polar Question Particles	50.06
105A	Ditransitive Constructions: The Verb 'Give'	50.00
78A	Coding of Evidentiality	49.64
118A	Predicative Adjectives	49.61
104A	Order of Person Markers on the Verb	48.80
4A	Voicing in Plosives and Fricatives	48.76
103A	Third Person Zero of Verbal Person Marking	48.14
113A	Symmetric and Asymmetric Standard Negation	46.78
34A	Occurrence of Nominal Plurality	45.70
17A	Rhythm Types	44.41
37A	Definite Articles	43.62
14A	Fixed Stress Locations	43.49
26A	Prefixing vs. Suffixing in Inflectional Morphology	41.94
3A	Consonant-Vowel Ratio	41.39
143A	Order of Negative Morpheme and Verb	40.51
71A	The Prohibitive	38.70
114A	Subtypes of Asymmetric Standard Negation	38.64
38A	Indefinite Articles	37.52
53A	Ordinal Numerals	36.77
1A	Consonant Inventories	35.77
144A	Position of Negative Word With Respect to Subject, Object, and Verb	29.63

Table S.2: Complete list of features ranked by vertical stability parameter  $v_i$  of Autologistic-Basic. The arithmetic means of the posterior samples are followed by the 95% highest posterior density (HPD) credible intervals (assuming unimodality).

Feature	$v_i$	95% HPD
73A The Optative	0.0281	[0.0086, 0.0462]
33A Coding of Nominal Plurality	0.0280	[0.0105, 0.0322]
6A Uvular Consonants	0.0278	[0.0106, 0.0455]
97A Relationship between the Order of Object and Verb and the Order of Adjective and Noun	0.0251	[0.0218, 0.0286]
86A Order of Genitive and Noun	0.0243	[0.0192, 0.0299]
95A Relationship between the Order of Object and Verb and the Order of Adposition and Noun Phrase	0.0237	[0.0204, 0.0280]
85A Order of Adposition and Noun Phrase	0.0228	[0.0188, 0.0266]
83A Order of Object and Verb	0.0227	[0.0193, 0.0264]
81A Order of Subject, Object and Verb	0.0217	[0.0178, 0.0247]
7A Glottalized Consonants	0.0216	[0.0105, 0.0322]
94A Order of Adverbial Subordinator and Clause	0.0204	[0.0149, 0.0264]
143E Preverbal Negative Morphemes	0.0201	[0.0161, 0.0239]
96A Relationship between the Order of Object and Verb and the Order of Relative Clause and Noun	0.0198	[0.0165, 0.0231]
143A Order of Negative Morpheme and Verb	0.0191	[0.0152, 0.0229]
77A Semantic Distinctions of Evidentiality	0.0188	[0.0131, 0.0263]
84A Order of Object, Oblique, and Verb	0.0184	[0.0153, 0.0231]
144A Position of Negative Word With Respect to Subject, Object, and Verb	0.0184	[0.0145, 0.0225]
78A Coding of Evidentiality	0.0183	[0.0115, 0.0246]
87A Order of Adjective and Noun	0.0183	[0.0145, 0.0216]
57A Position of Pronominal Possessive Affixes	0.0175	[0.0142, 0.0201]
90A Order of Relative Clause and Noun	0.0168	[0.0108, 0.0213]
26A Prefixing vs. Suffixing in Inflectional Morphology	0.0167	[0.0114, 0.0219]
53A Ordinal Numerals	0.0165	[0.0116, 0.0224]
51A Position of Case Affixes	0.0164	[0.0133, 0.0194]
89A Order of Numeral and Noun	0.0147	[0.0110, 0.0181]
4A Voicing in Plosives and Fricatives	0.0145	[0.0071, 0.0209]
38A Indefinite Articles	0.0141	[0.0065, 0.0210]
90C Postnominal relative clauses	0.0138	[0.0056, 0.0239]

Table S.2: (continued)

	Feature	$v_i$	95% HPD
129A	Hand and Arm	0.0136	[0.0060, 0.0212]
88A	Order of Demonstrative and Noun	0.0136	[0.0112, 0.0162]
143G	Minor morphological means of signaling negation	0.0133	[0.0017, 0.0283]
143F	Postverbal Negative Morphemes	0.0133	[0.0104, 0.0162]
124A	'Want' Complement Subjects	0.0127	[0.0068, 0.0186]
37A	Definite Articles	0.0126	[0.0053, 0.0183]
5A	Voicing and Gaps in Plosive Systems	0.0123	[0.0073, 0.0164]
93A	Position of Interrogative Phrases in Content Questions	0.0123	[0.0088, 0.0165]
9A	The Velar Nasal	0.0123	[0.0040, 0.0255]
71A	The Prohibitive	0.0122	[0.0044, 0.0187]
118A	Predicative Adjectives	0.0122	[0.0026, 0.0207]
112A	Negative Morphemes	0.0121	[0.0081, 0.0153]
103A	Third Person Zero of Verbal Person Marking	0.0117	[0.0061, 0.0177]
111A	Nonperiphrastic Causative Constructions	0.0113	[0.0037, 0.0198]
113A	Symmetric and Asymmetric Standard Negation	0.0113	[0.0040, 0.0178]
17A	Rhythm Types	0.0111	[0.0059, 0.0159]
91A	Order of Degree Word and Adjective	0.0107	[0.0046, 0.0169]
1A	Consonant Inventories	0.0103	[0.0024, 0.0160]
114A	Subtypes of Asymmetric Standard Negation	0.0103	[0.0021, 0.0173]
120A	Zero Copula for Predicate Nominals	0.0102	[0.0040, 0.0165]
12A	Syllable Structure	0.0095	[0.0026, 0.0149]
100A	Alignment of Verbal Person Marking	0.0095	[0.0045, 0.0146]
13A	Tone	0.0091	[0.0031, 0.0138]
34A	Occurrence of Nominal Plurality	0.0089	[0.0020, 0.0145]
16A	Weight Factors in Weight-Sensitive Stress Systems	0.0089	[0.0042, 0.0137]
104A	Order of Person Markers on the Verb	0.0085	[0.0032, 0.0145]
64A	Nominal and Verbal Conjunction	0.0083	[0.0012, 0.0140]
2A	Vowel Quality Inventories	0.0083	[0.0030, 0.0140]
82A	Order of Subject and Verb	0.0082	[0.0049, 0.0113]
3A	Consonant-Vowel Ratio	0.0080	[0.0019, 0.0133]
52A	Comitatives and Instrumentals	0.0078	[0.0022, 0.0124]
15A	Weight-Sensitive Stress	0.0074	[0.0032, 0.0111]



Table S.2: (continued)

	Feature	$v_i$	95% HPD
14A	Fixed Stress Locations	0.0074	[0.0017, 0.0128]
144B	Position of negative words relative to beginning and end of clause and with respect to adjacency to verb	0.0072	[0.0038, 0.0117]
119A	Nominal and Locational Predication	0.0071	[0.0027, 0.0118]
8A	Lateral Consonants	0.0071	[0.0019, 0.0120]
27A	Reduplication	0.0071	[0.0013, 0.0129]
92A	Position of Polar Question Particles	0.0071	[0.0014, 0.0124]
107A	Passive Constructions	0.0070	[0.0016, 0.0133]
102A	Verbal Person Marking	0.0061	[0.0007, 0.0117]
69A	Position of Tense-Aspect Affixes	0.0059	[0.0018, 0.0109]
44A	Gender Distinctions in Independent Personal Pronouns	0.0058	[0.0007, 0.0104]
116A	Polar Questions	0.0055	[0.0017, 0.0095]
55A	Numeral Classifiers	0.0051	[0.0010, 0.0096]
48A	Person Marking on Adpositions	0.0051	[0.0004, 0.0093]
11A	Front Rounded Vowels	0.0045	[0.0006, 0.0099]
19A	Presence of Uncommon Consonants	0.0044	[0.0005, 0.0088]
70A	The Morphological Imperative	0.0043	[0.0006, 0.0082]
46A	Indefinite Pronouns	0.0035	[0.0003, 0.0076]
101A	Expression of Pronominal Subjects	0.0035	[0.0010, 0.0064]
105A	Ditransitive Constructions: The Verb 'Give'	0.0034	[0.0002, 0.0066]
130A	Finger and Hand	0.0034	[0.0003, 0.0072]
72A	Imperative-Hortative Systems	0.0030	[0.0002, 0.0064]
18A	Absence of Common Consonants	0.0024	[0.0001, 0.0056]

Table S.3: Complete list of features ranked by horizontal diffusibility parameter  $h_i$  of Autologistic-Basic. The arithmetic means of the posterior samples are followed by the 95% highest posterior density (HPD) credible intervals (assuming unimodality).

	Feature	$h_i$	95% HPD
144A	Position of Negative Word With Respect to Subject, Object, and Verb	0.0222	[0.0146, 0.0289]
97A	Relationship between the Order of Object and Verb and the Order of Adjective and Noun	0.0182	[0.0164, 0.0204]
93A	Position of Interrogative Phrases in Content Questions	0.0166	[0.0150, 0.0184]
81A	Order of Subject, Object and Verb	0.0163	[0.0145, 0.0176]
82A	Order of Subject and Verb	0.0163	[0.0115, 0.0201]
95A	Relationship between the Order of Object and Verb and the Order of Adposition and Noun Phrase	0.0159	[0.0140, 0.0175]
83A	Order of Object and Verb	0.0152	[0.0137, 0.0167]
89A	Order of Numeral and Noun	0.0150	[0.0132, 0.0167]
92A	Position of Polar Question Particles	0.0144	[0.0120, 0.0168]
85A	Order of Adposition and Noun Phrase	0.0144	[0.0120, 0.0161]
86A	Order of Genitive and Noun	0.0140	[0.0127, 0.0158]
107A	Passive Constructions	0.0140	[0.0109, 0.0172]
57A	Position of Pronominal Possessive Affixes	0.0138	[0.0110, 0.0163]
51A	Position of Case Affixes	0.0135	[0.0110, 0.0163]
6A	Uvular Consonants	0.0135	[0.0066, 0.0187]
96A	Relationship between the Order of Object and Verb and the Order of Relative Clause and Noun	0.0134	[0.0104, 0.0168]
88A	Order of Demonstrative and Noun	0.0134	[0.0118, 0.0148]
1A	Consonant Inventories	0.0125	[0.0058, 0.0167]
118A	Predicative Adjectives	0.0120	[0.0065, 0.0158]
4A	Voicing in Plosives and Fricatives	0.0120	[0.0097, 0.0141]
78A	Coding of Evidentiality	0.0107	[0.0071, 0.0137]
91A	Order of Degree Word and Adjective	0.0106	[0.0084, 0.0127]
87A	Order of Adjective and Noun	0.0106	[0.0088, 0.0123]
77A	Semantic Distinctions of Evidentiality	0.0103	[0.0077, 0.0133]
120A	Zero Copula for Predicate Nominals	0.0102	[0.0026, 0.0145]
37A	Definite Articles	0.0102	[0.0054, 0.0142]
38A	Indefinite Articles	0.0102	[0.0069, 0.0139]
9A	The Velar Nasal	0.0101	[0.0068, 0.0126]

Table S.3: (continued)

	Feature	$h_i$	95% HPD
26A	Prefixing vs. Suffixing in Inflectional Morphology	0.0099	[0.0069, 0.0133]
112A	Negative Morphemes	0.0098	[0.0072, 0.0126]
119A	Nominal and Locational Predication	0.0097	[0.0070, 0.0126]
5A	Voicing and Gaps in Plosive Systems	0.0096	[0.0076, 0.0118]
27A	Reduplication	0.0092	[0.0032, 0.0145]
2A	Vowel Quality Inventories	0.0090	[0.0061, 0.0119]
143A	Order of Negative Morpheme and Verb	0.0089	[0.0058, 0.0114]
19A	Presence of Uncommon Consonants	0.0085	[0.0046, 0.0122]
143F	Postverbal Negative Morphemes	0.0084	[0.0056, 0.0110]
69A	Position of Tense-Aspect Affixes	0.0084	[0.0065, 0.0107]
71A	The Prohibitive	0.0084	[0.0039, 0.0122]
129A	Hand and Arm	0.0083	[0.0042, 0.0115]
7A	Glottalized Consonants	0.0080	[0.0025, 0.0126]
11A	Front Rounded Vowels	0.0080	[0.0036, 0.0119]
143E	Preverbal Negative Morphemes	0.0079	[0.0051, 0.0102]
84A	Order of Object, Oblique, and Verb	0.0074	[0.0020, 0.0122]
12A	Syllable Structure	0.0074	[0.0035, 0.0105]
103A	Third Person Zero of Verbal Person Marking	0.0074	[0.0030, 0.0124]
55A	Numeral Classifiers	0.0071	[0.0031, 0.0114]
52A	Comitatives and Instrumentals	0.0071	[0.0029, 0.0111]
73A	The Optative	0.0070	[0.0038, 0.0107]
101A	Expression of Pronominal Subjects	0.0066	[0.0043, 0.0083]
113A	Symmetric and Asymmetric Standard Negation	0.0060	[0.0014, 0.0104]
17A	Rhythm Types	0.0059	[0.0018, 0.0094]
64A	Nominal and Verbal Conjunction	0.0058	[0.0013, 0.0104]
53A	Ordinal Numerals	0.0056	[0.0008, 0.0098]
16A	Weight Factors in Weight-Sensitive Stress Systems	0.0055	[0.0031, 0.0085]
102A	Verbal Person Marking	0.0055	[0.0016, 0.0086]
46A	Indefinite Pronouns	0.0053	[0.0008, 0.0102]
130A	Finger and Hand	0.0052	[0.0019, 0.0088]
3A	Consonant-Vowel Ratio	0.0049	[0.0012, 0.0086]
15A	Weight-Sensitive Stress	0.0048	[0.0020, 0.0072]

Table S.3: (continued)

	Feature	$h_i$	95% HPD
100A	Alignment of Verbal Person Marking	0.0048	[0.0021, 0.0089]
14A	Fixed Stress Locations	0.0047	[0.0009, 0.0085]
70A	The Morphological Imperative	0.0046	[0.0013, 0.0092]
124A	'Want' Complement Subjects	0.0045	[0.0009, 0.0082]
48A	Person Marking on Adpositions	0.0045	[0.0015, 0.0082]
44A	Gender Distinctions in Independent Personal Pronouns	0.0043	[0.0017, 0.0068]
90A	Order of Relative Clause and Noun	0.0043	[0.0017, 0.0067]
116A	Polar Questions	0.0040	[0.0018, 0.0062]
90C	Postnominal relative clauses	0.0039	[0.0002, 0.0085]
33A	Coding of Nominal Plurality	0.0039	[0.0012, 0.0063]
104A	Order of Person Markers on the Verb	0.0038	[0.0005, 0.0076]
72A	Imperative-Hortative Systems	0.0038	[0.0008, 0.0072]
105A	Ditransitive Constructions: The Verb 'Give'	0.0037	[0.0007, 0.0071]
114A	Subtypes of Asymmetric Standard Negation	0.0037	[0.0006, 0.0080]
143G	Minor morphological means of signaling negation	0.0035	[0.0003, 0.0076]
144B	Position of negative words relative to beginning and end of clause and with respect to adjacency to verb	0.0030	[0.0003, 0.0056]
13A	Tone	0.0025	[0.0003, 0.0049]
34A	Occurrence of Nominal Plurality	0.0024	[0.0001, 0.0053]
8A	Lateral Consonants	0.0023	[0.0002, 0.0042]
94A	Order of Adverbial Subordinator and Clause	0.0022	[0.0001, 0.0050]
18A	Absence of Common Consonants	0.0019	[0.0002, 0.0040]
111A	Nonperiphrastic Causative Constructions	0.0013	[0.0001, 0.0027]

Table S.4: Complete list of features ranked by vertical stability parameter  $v_i$  of Autologistic-Disjoint. The arithmetic means of the posterior samples are followed by the 95% highest posterior density (HPD) credible intervals (assuming unimodality).

	Feature	$v_i$	95% HPD
73A	The Optative	0.0348	[0.0093, 0.0620]
6A	Uvular Consonants	0.0285	[0.0113, 0.0485]
33A	Coding of Nominal Plurality	0.0281	[0.0054, 0.0386]
7A	Glottalized Consonants	0.0270	[0.0092, 0.0443]
143G	Minor morphological means of signaling negation	0.0218	[0.0004, 0.0472]
78A	Coding of Evidentiality	0.0169	[0.0061, 0.0278]
94A	Order of Adverbial Subordinator and Clause	0.0161	[0.0069, 0.0237]
111A	Nonperiphrastic Causative Constructions	0.0159	[0.0065, 0.0266]
84A	Order of Object, Oblique, and Verb	0.0158	[0.0065, 0.0243]
124A	'Want' Complement Subjects	0.0154	[0.0076, 0.0228]
86A	Order of Genitive and Noun	0.0154	[0.0093, 0.0224]
77A	Semantic Distinctions of Evidentiality	0.0152	[0.0057, 0.0267]
4A	Voicing in Plosives and Fricatives	0.0142	[0.0041, 0.0236]
96A	Relationship between the Order of Object and Verb and the Order of Relative Clause and Noun	0.0140	[0.0075, 0.0189]
38A	Indefinite Articles	0.0138	[0.0047, 0.0241]
90C	Postnominal relative clauses	0.0135	[0.0024, 0.0248]
9A	The Velar Nasal	0.0132	[0.0003, 0.0332]
93A	Position of Interrogative Phrases in Content Questions	0.0122	[0.0066, 0.0193]
118A	Predicative Adjectives	0.0122	[0.0004, 0.0206]
90A	Order of Relative Clause and Noun	0.0120	[0.0056, 0.0189]
37A	Definite Articles	0.0118	[0.0020, 0.0217]
103A	Third Person Zero of Verbal Person Marking	0.0113	[0.0042, 0.0198]
95A	Relationship between the Order of Object and Verb and the Order of Adposition and Noun Phrase	0.0112	[0.0057, 0.0177]
26A	Prefixing vs. Suffixing in Inflectional Morphology	0.0110	[0.0031, 0.0202]
52A	Comitatives and Instrumentals	0.0109	[0.0037, 0.0183]
88A	Order of Demonstrative and Noun	0.0105	[0.0053, 0.0161]
97A	Relationship between the Order of Object and Verb and the Order of Adjective and Noun	0.0102	[0.0043, 0.0148]
71A	The Prohibitive	0.0101	[0.0017, 0.0183]

Table S.4: (continued)

	Feature	$v_i$	95% HPD
51A	Position of Case Affixes	0.0101	[0.0042, 0.0164]
120A	Zero Copula for Predicate Nominals	0.0100	[0.0024, 0.0184]
5A	Voicing and Gaps in Plosive Systems	0.0100	[0.0019, 0.0182]
13A	Tone	0.0092	[0.0030, 0.0157]
17A	Rhythm Types	0.0092	[0.0019, 0.0155]
85A	Order of Adposition and Noun Phrase	0.0091	[0.0033, 0.0150]
1A	Consonant Inventories	0.0090	[0.0009, 0.0195]
27A	Reduplication	0.0088	[0.0017, 0.0152]
129A	Hand and Arm	0.0086	[0.0001, 0.0189]
34A	Occurrence of Nominal Plurality	0.0086	[0.0019, 0.0165]
113A	Symmetric and Asymmetric Standard Negation	0.0085	[0.0006, 0.0181]
102A	Verbal Person Marking	0.0081	[0.0014, 0.0157]
64A	Nominal and Verbal Conjunction	0.0081	[0.0009, 0.0150]
114A	Subtypes of Asymmetric Standard Negation	0.0081	[0.0005, 0.0159]
100A	Alignment of Verbal Person Marking	0.0080	[0.0015, 0.0143]
53A	Ordinal Numerals	0.0078	[0.0005, 0.0155]
91A	Order of Degree Word and Adjective	0.0074	[0.0007, 0.0143]
143E	Preverbal Negative Morphemes	0.0074	[0.0023, 0.0123]
81A	Order of Subject, Object and Verb	0.0073	[0.0021, 0.0123]
2A	Vowel Quality Inventories	0.0072	[0.0009, 0.0153]
143A	Order of Negative Morpheme and Verb	0.0068	[0.0025, 0.0122]
83A	Order of Object and Verb	0.0067	[0.0020, 0.0119]
104A	Order of Person Markers on the Verb	0.0066	[0.0005, 0.0133]
3A	Consonant-Vowel Ratio	0.0066	[0.0004, 0.0136]
107A	Passive Constructions	0.0065	[0.0009, 0.0121]
87A	Order of Adjective and Noun	0.0064	[0.0015, 0.0109]
57A	Position of Pronominal Possessive Affixes	0.0060	[0.0008, 0.0110]
55A	Numeral Classifiers	0.0060	[0.0007, 0.0114]
48A	Person Marking on Adpositions	0.0060	[0.0006, 0.0118]
11A	Front Rounded Vowels	0.0057	[0.0003, 0.0121]
14A	Fixed Stress Locations	0.0057	[0.0004, 0.0115]
19A	Presence of Uncommon Consonants	0.0056	[0.0005, 0.0105]

Table S.4: (continued)

	Feature	$v_i$	95% HPD
44A	Gender Distinctions in Independent Personal Pronouns	0.0056	[0.0006, 0.0112]
144A	Position of Negative Word With Respect to Subject, Object, and Verb	0.0055	[0.0004, 0.0115]
16A	Weight Factors in Weight-Sensitive Stress Systems	0.0054	[0.0007, 0.0108]
105A	Ditransitive Constructions: The Verb 'Give'	0.0050	[0.0004, 0.0101]
15A	Weight-Sensitive Stress	0.0050	[0.0007, 0.0103]
119A	Nominal and Locational Predication	0.0050	[0.0003, 0.0098]
70A	The Morphological Imperative	0.0050	[0.0003, 0.0100]
12A	Syllable Structure	0.0050	[0.0004, 0.0105]
144B	Position of negative words relative to beginning and end of clause and with respect to adjacency to verb	0.0050	[0.0006, 0.0094]
69A	Position of Tense-Aspect Affixes	0.0048	[0.0006, 0.0097]
72A	Imperative-Hortative Systems	0.0046	[0.0007, 0.0104]
46A	Indefinite Pronouns	0.0046	[0.0003, 0.0095]
116A	Polar Questions	0.0046	[0.0006, 0.0087]
92A	Position of Polar Question Particles	0.0045	[0.0002, 0.0096]
143F	Postverbal Negative Morphemes	0.0042	[0.0007, 0.0084]
89A	Order of Numeral and Noun	0.0038	[0.0003, 0.0077]
112A	Negative Morphemes	0.0038	[0.0005, 0.0076]
8A	Lateral Consonants	0.0035	[0.0005, 0.0074]
101A	Expression of Pronominal Subjects	0.0032	[0.0002, 0.0068]
130A	Finger and Hand	0.0030	[0.0001, 0.0069]
82A	Order of Subject and Verb	0.0029	[0.0005, 0.0059]
18A	Absence of Common Consonants	0.0029	[0.0002, 0.0066]

Table S.5: Complete list of features ranked by horizontal diffusibility parameter  $h_i$  of Autologistic-Disjoint. The arithmetic means of the posterior samples are followed by the 95% highest posterior density (HPD) credible intervals (assuming unimodality).

	Feature	$h_i$	95% HPD
144A	Position of Negative Word With Respect to Subject, Object, and Verb	0.0202	[0.0149, 0.0255]
93A	Position of Interrogative Phrases in Content Questions	0.0165	[0.0140, 0.0188]
97A	Relationship between the Order of Object and Verb and the Order of Adjective and Noun	0.0158	[0.0136, 0.0176]
82A	Order of Subject and Verb	0.0153	[0.0090, 0.0189]
81A	Order of Subject, Object and Verb	0.0141	[0.0123, 0.0160]
95A	Relationship between the Order of Object and Verb and the Order of Adposition and Noun Phrase	0.0141	[0.0121, 0.0161]
6A	Uvular Consonants	0.0139	[0.0052, 0.0199]
92A	Position of Polar Question Particles	0.0137	[0.0109, 0.0168]
107A	Passive Constructions	0.0132	[0.0088, 0.0169]
96A	Relationship between the Order of Object and Verb and the Order of Relative Clause and Noun	0.0129	[0.0090, 0.0168]
83A	Order of Object and Verb	0.0129	[0.0109, 0.0146]
88A	Order of Demonstrative and Noun	0.0127	[0.0105, 0.0147]
85A	Order of Adposition and Noun Phrase	0.0126	[0.0106, 0.0146]
51A	Position of Case Affixes	0.0125	[0.0096, 0.0157]
89A	Order of Numeral and Noun	0.0124	[0.0105, 0.0143]
4A	Voicing in Plosives and Fricatives	0.0120	[0.0092, 0.0144]
57A	Position of Pronominal Possessive Affixes	0.0118	[0.0088, 0.0146]
118A	Predicative Adjectives	0.0113	[0.0064, 0.0145]
78A	Coding of Evidentiality	0.0112	[0.0081, 0.0148]
86A	Order of Genitive and Noun	0.0112	[0.0093, 0.0128]
11A	Front Rounded Vowels	0.0106	[0.0052, 0.0166]
27A	Reduplication	0.0106	[0.0040, 0.0161]
37A	Definite Articles	0.0102	[0.0047, 0.0148]
77A	Semantic Distinctions of Evidentiality	0.0102	[0.0067, 0.0131]
120A	Zero Copula for Predicate Nominals	0.0102	[0.0058, 0.0148]
1A	Consonant Inventories	0.0098	[0.0021, 0.0173]
38A	Indefinite Articles	0.0098	[0.0056, 0.0134]
9A	The Velar Nasal	0.0098	[0.0055, 0.0136]



Table S.5: (continued)

	Feature	$h_i$	95% HPD
52A	Comitatives and Instrumentals	0.0097	[0.0053, 0.0134]
91A	Order of Degree Word and Adjective	0.0092	[0.0061, 0.0116]
2A	Vowel Quality Inventories	0.0092	[0.0069, 0.0120]
5A	Voicing and Gaps in Plosive Systems	0.0089	[0.0066, 0.0114]
84A	Order of Object, Oblique, and Verb	0.0088	[0.0052, 0.0126]
26A	Prefixing vs. Suffixing in Inflectional Morphology	0.0087	[0.0052, 0.0115]
55A	Numeral Classifiers	0.0087	[0.0050, 0.0133]
87A	Order of Adjective and Noun	0.0087	[0.0071, 0.0104]
7A	Glottalized Consonants	0.0086	[0.0032, 0.0120]
71A	The Prohibitive	0.0085	[0.0039, 0.0131]
112A	Negative Morphemes	0.0083	[0.0065, 0.0105]
19A	Presence of Uncommon Consonants	0.0082	[0.0051, 0.0110]
69A	Position of Tense-Aspect Affixes	0.0081	[0.0058, 0.0102]
119A	Nominal and Locational Predication	0.0077	[0.0043, 0.0110]
143A	Order of Negative Morpheme and Verb	0.0077	[0.0051, 0.0106]
143F	Postverbal Negative Morphemes	0.0073	[0.0038, 0.0101]
102A	Verbal Person Marking	0.0071	[0.0039, 0.0103]
64A	Nominal and Verbal Conjunction	0.0070	[0.0028, 0.0108]
103A	Third Person Zero of Verbal Person Marking	0.0070	[0.0023, 0.0101]
12A	Syllable Structure	0.0069	[0.0039, 0.0100]
17A	Rhythm Types	0.0067	[0.0024, 0.0104]
129A	Hand and Arm	0.0064	[0.0014, 0.0113]
46A	Indefinite Pronouns	0.0064	[0.0026, 0.0096]
72A	Imperative-Hortative Systems	0.0061	[0.0025, 0.0092]
143E	Preverbal Negative Morphemes	0.0060	[0.0033, 0.0088]
113A	Symmetric and Asymmetric Standard Negation	0.0060	[0.0011, 0.0099]
73A	The Optative	0.0058	[0.0021, 0.0089]
105A	Ditransitive Constructions: The Verb 'Give'	0.0058	[0.0021, 0.0100]
130A	Finger and Hand	0.0056	[0.0019, 0.0097]
3A	Consonant-Vowel Ratio	0.0055	[0.0017, 0.0089]
124A	'Want' Complement Subjects	0.0055	[0.0011, 0.0099]
90A	Order of Relative Clause and Noun	0.0054	[0.0006, 0.0091]

Table S.5: (continued)

	Feature	$h_i$	95% HPD
48A	Person Marking on Adpositions	0.0053	[0.0017, 0.0085]
70A	The Morphological Imperative	0.0052	[0.0022, 0.0081]
101A	Expression of Pronominal Subjects	0.0050	[0.0020, 0.0083]
116A	Polar Questions	0.0048	[0.0025, 0.0076]
14A	Fixed Stress Locations	0.0045	[0.0011, 0.0084]
44A	Gender Distinctions in Independent Personal Pronouns	0.0045	[0.0015, 0.0070]
16A	Weight Factors in Weight-Sensitive Stress Systems	0.0044	[0.0007, 0.0080]
33A	Coding of Nominal Plurality	0.0043	[0.0015, 0.0072]
15A	Weight-Sensitive Stress	0.0043	[0.0014, 0.0082]
114A	Subtypes of Asymmetric Standard Negation	0.0035	[0.0003, 0.0067]
100A	Alignment of Verbal Person Marking	0.0035	[0.0007, 0.0068]
53A	Ordinal Numerals	0.0035	[0.0004, 0.0069]
143G	Minor morphological means of signaling negation	0.0033	[0.0004, 0.0071]
111A	Nonperiphrastic Causative Constructions	0.0028	[0.0003, 0.0058]
18A	Absence of Common Consonants	0.0028	[0.0007, 0.0056]
13A	Tone	0.0027	[0.0005, 0.0054]
144B	Position of negative words relative to beginning and end of clause and with respect to adjacency to verb	0.0027	[0.0003, 0.0051]
90C	Postnominal relative clauses	0.0026	[0.0004, 0.0052]
104A	Order of Person Markers on the Verb	0.0026	[0.0005, 0.0056]
34A	Occurrence of Nominal Plurality	0.0020	[0.0002, 0.0044]
94A	Order of Adverbial Subordinator and Clause	0.0018	[0.0000, 0.0044]
8A	Lateral Consonants	0.0016	[0.0001, 0.0033]

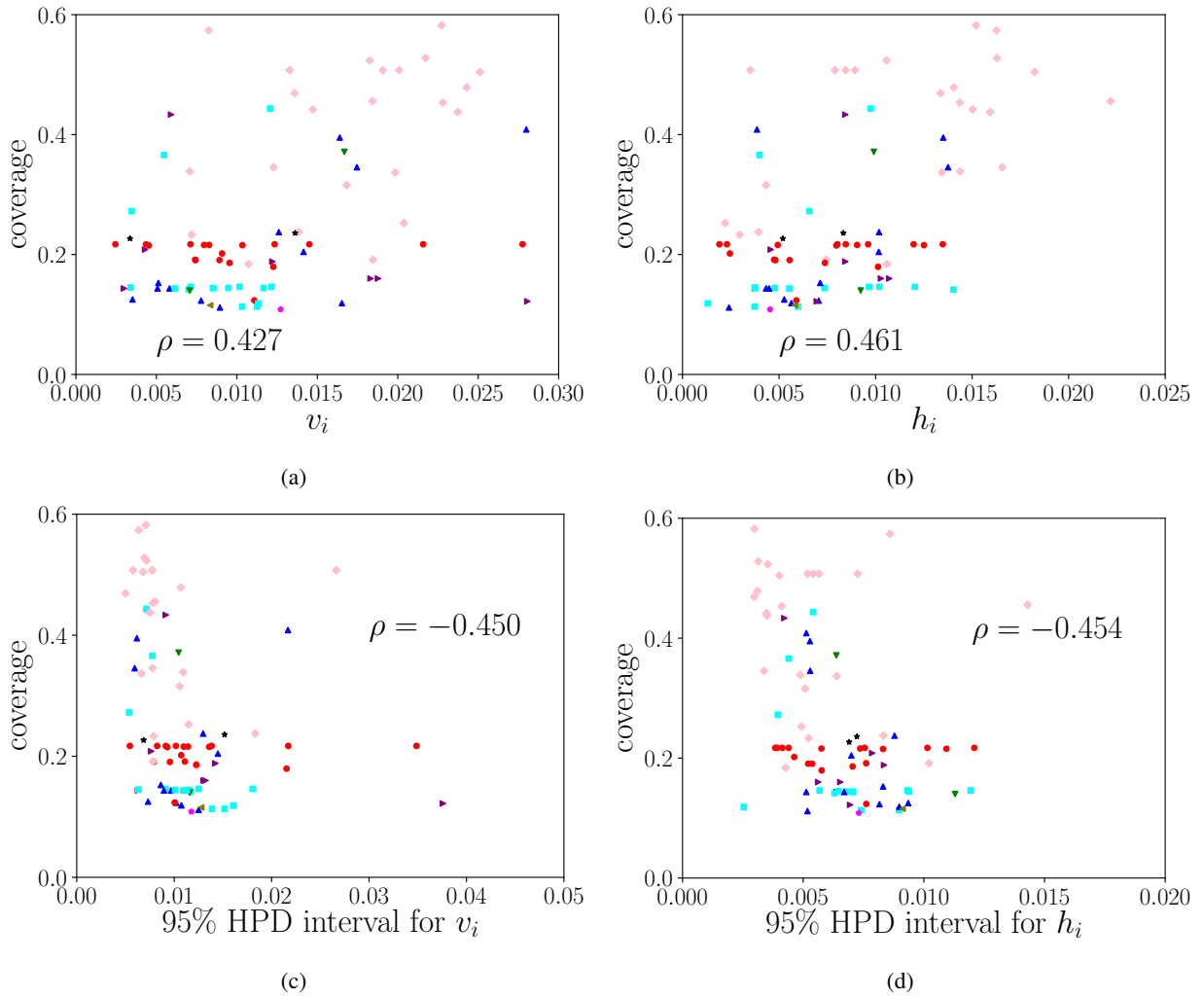
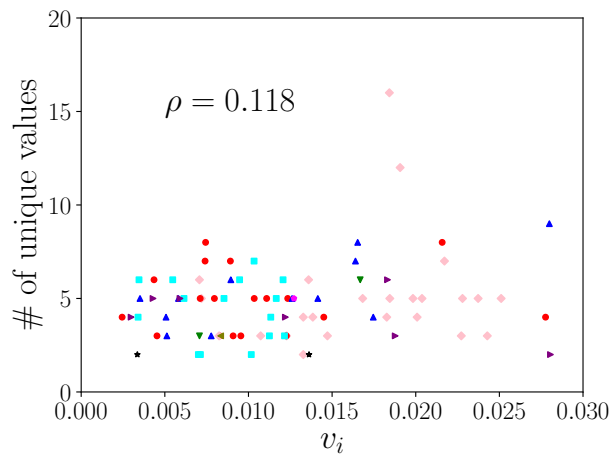
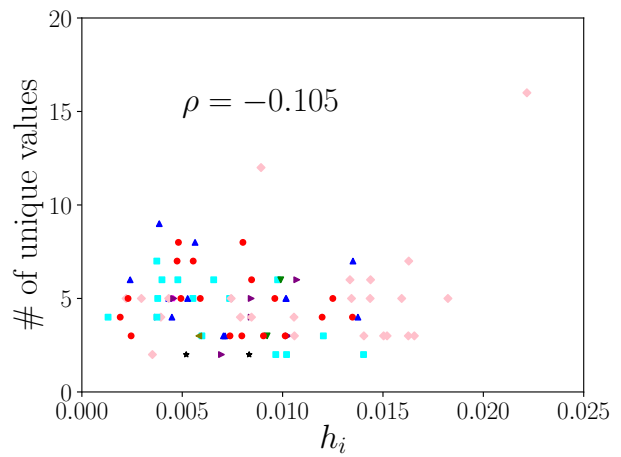


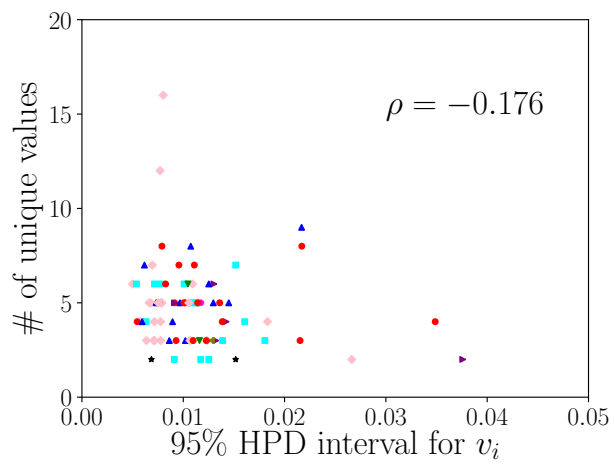
Figure S.1: Correlations between coverage and various variables of Autologistic-Basic. Coverage is the ratio of languages whose feature values are present.  $v_i$  and  $h_i$  are the arithmetic means of the posterior samples. HDP refers to the 95% highest posterior density (HPD) credible intervals.



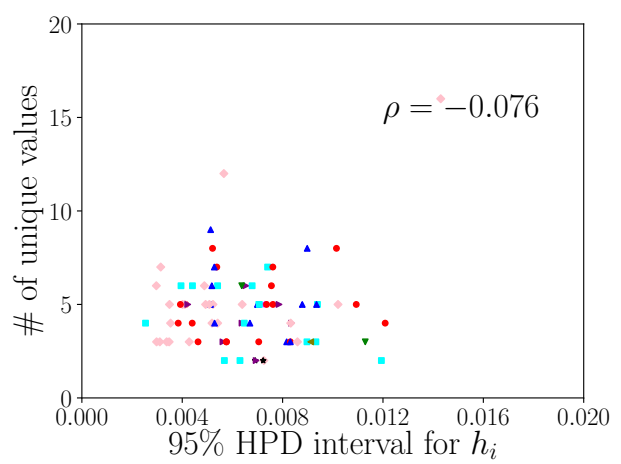
(a)



(b)



(c)



(d)

Figure S.2: Correlations between the number of unique values for each feature and various variables of Autologistic-Basic.

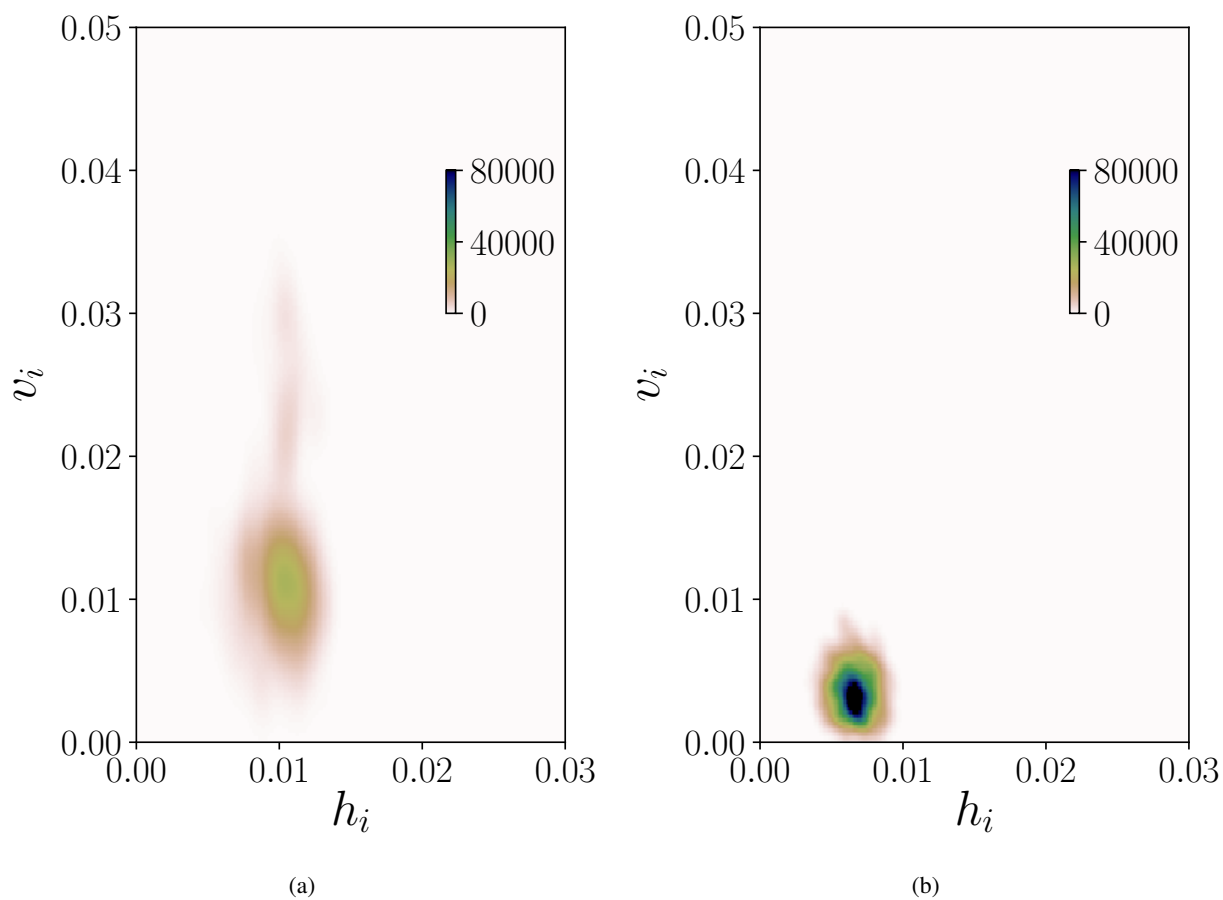


Figure S.3: Two examples of the posterior distribution of  $v_i$  and  $h_i$  of Autologistic-Basic. (a) Feature 9A “The Velar Nasal.” This feature exhibits relatively high uncertainty. (b) Feature 101A “Expression of Pronominal Subjects.” A feature with relatively small uncertainty about the parameters.

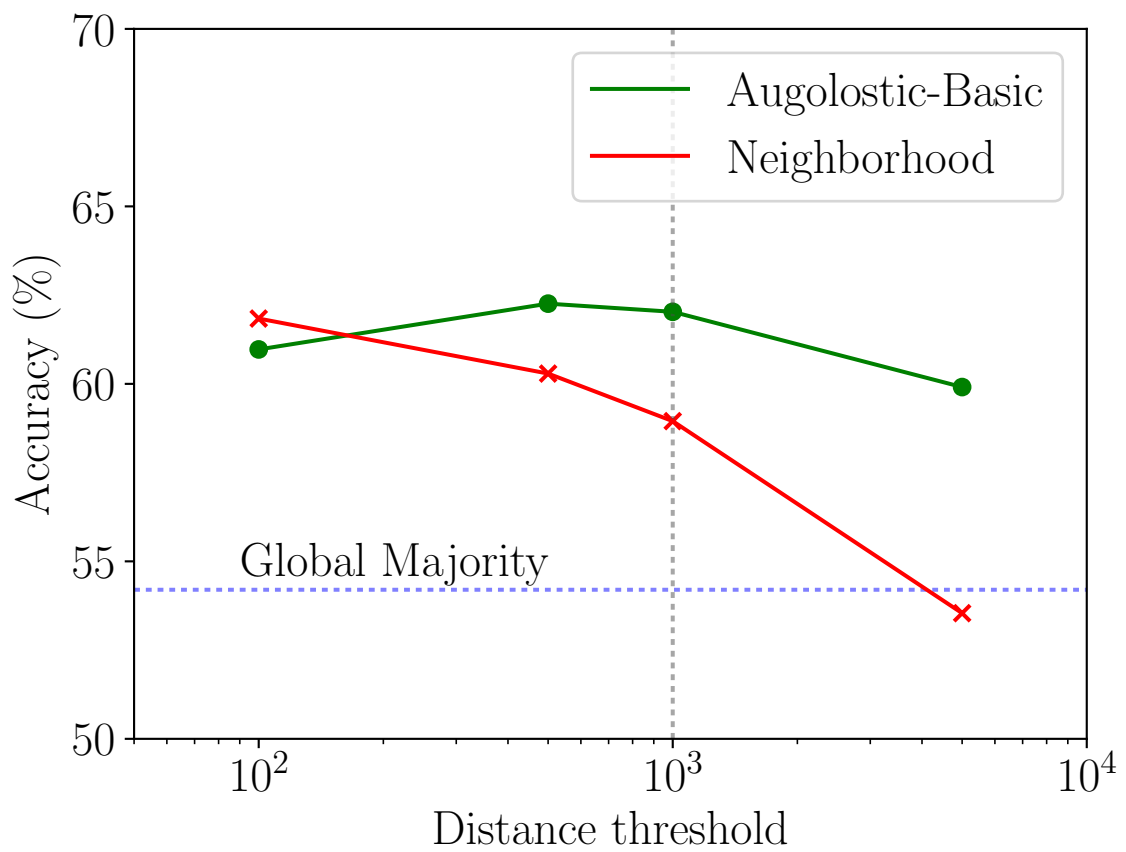


Figure S.4: The relationship between the distance threshold of the spatial neighbor graph and microaveraged accuracy of missing value imputation.